“DYNAMIC RESPONSE OF BEAM AND FRAME STRUCTURE SUBJECTED TO MOVING LOAD”

A thesis submitted in the partial fulfillment of the requirements for the degree of

MASTER OF TECHNOLOGY
IN
MECHANICAL ENGINEERING

(Specialization: Machine Design and Analysis)

Submitted by

JITENDRA NAIK
(ROLL NO: 212ME1277)

DEPARTMENT OF MECHANICAL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA-769008, INDIA

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CERTIFICATE

This is to certify that the thesis entitled, "**DYNAMIC RESPONSE OF BEAM AND FRAME STRUCTURE SUBJECTED TO MOVING LOAD**" submitted by Mr.**JITENDRA NAIK (212ME1277)** in partial fulfillment of the requirements for the award of **Master Of Technology** degree in **Mechanical Engineering** with specialization in **Machine Design and Analysis** at the National Institute of Technology, Rourkela (India) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

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Contents

ABSTRACT

In this work the dynamic response of beam and the frame structure is studied. Analysis is done with the help of finite element analysis method. Beam structure is divided into ten elements for constituting the mass and stiffness matrices. Frame structure consists of two columns of equal geometry. Each column is divided into ten elements having the same property as beam. Euler Bernoulli beam theory is assumed for the analysis of these structures. Three different boundary conditions are taken such as clamped-clamped, pinned-pinned and clamped-pinned for dynamic analysis. Two degree of freedom is considered for beam and the effect of moment and inertia is neglected for moving load problem. A concentrated load moving at constant speed over the beam. Comparison between static and dynamic displacement is done for different boundary conditions. MATLAB programs are developed to get the response and the results were validated with previous research work. The study of dynamic response is having a great importance in the field of mechanical, civil and aerospace engineering. A dynamic load varies with time and space for example railway track, roadways, bridges, cranes, rope ways are some examples of structures subjected to dynamic load. The beam and the columns of the frame structure are divided into ten elements each. Mid point displacement is studied for concentrated point load moving at constant speed. The obtain results are collected and response graph are plotted for different boundary conditions. The results are compared and conclusions are drawn.

Keywords: beam, frame, Moving mass, dynamic response, Finite element method.

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NOMENCLATURE

E : modulus of elasticity(GPa)

I : moment of inertia

L : total length of the beam

ρ : density of the beam

A : area of cross section of beam

l : length of beam element

x, y : axial co-ordinates

x : position of the moving load from left end of the beam

N_i : shape function

y(x, t) : transverse displacement

M : overall mass matrix of beam

K : overall stiffness matrix of beam

T : transformation matrix

k1, k2 : spring stiffness

t : time

V : velocity of moving load

m : mass

τ : total time taken by moving load along the beam

ω : natural frequency

Δt : time interval

P : moving concentrated load

T_i : time period of ith natural frequency

Chapter 1

INTRODUCTION

Moving load is unpreventable in structural dynamics. Many research have been done on this problem during past years. Till the middle of the 19^{th} century the effect of dynamic load was unknown. The collapsed of Stephenson's bridge over Dee Chester river in London in 1947 attracted many researchers to study the response of structure subjected to moving load. Many researchers took interest in study the response of dynamic structure like Railway Bridge and Highway Bridge because it is the most common practical problem related to dynamic response. In all the dynamic problems, beam type structure is the most common type of structure used in the field of civil, mechanical and aerospace engineering. Bridge vehicle interaction is the most common problem in moving load analysis and it's been the vast area of research. If the speed of the vehicle is very slow, it could not be treated as moving load problem because at low speed it behaves as a static load condition. This problem can be solved by traditional ways. If the vehicle is moving at constant speed then it will be treated as moving load problem. This problem can be solve by mathematical and computational analysis. In the past years dynamic load problem are about the highway and railway bridge excited by moving vehicle. Almost all engineering structure is subjected to time and space varying load. Due to traffic intensity many factors arises which need to study the structure in many aspects. In this thesis the simple beam is investigated with different boundary condition. In addition to that the problem is extended to study the response of moving load on frame structure. Effect on the response with additional stiffness is investigated. The effect of inertia and moment are neglected for moving load analysis. Finite element method is approached to study the response of beam and frame structure.

Chapter 2

LITERATURE SURVEY

Dynamic response of beam is an essential topic of research in structural dynamics. Beam is a most common type of structure used in the field of aerospace engineering, mechanical engineering and civil engineering. Many of these structures subjected to dynamic load. Beam with moving load is a special topic of research in structural dynamics as compared to other load because it subjected to time and space varying load. Many approaches have been taken by scientists to solve the dynamic problem, hence a lots of literature exists

Thambiratnam & Zhuge [1] investigated a simple model to study the behavior of simply supported beam. The model was supported on an elastic foundation. In the first case the beam was analyze for static condition and the results obtain from the analysis were applied for the analysis of railway track structure. Same model was tested for free vibration condition. The problem is formulated for the beam of any length and it is very useful for the analysis of railway track structure. They prolonged their experiment to the analysis of beam having elastic foundation subjected to moving load. All these experiment give rise an approach to for free vibration, static and dynamic analysis of beam supported on an elastic foundation. This procedure simplified the complexity of the problem because the axial effect and the effect of the change in beam properties, beam length be easily accommodated in this problem. The effect of stiffness on foundation was investigated, travelling speed and beam length on dynamic magnification factor.

Serdar hugul [2] investigated the dynamic response of systems subjected to moving load. The system includes the beam structure with different boundary condition and the frame structure. He solved the dynamic problem by two numerical method i.e. the Finite element method and the Newmark integration method. For the analysis of forced vibration Newmark integration is employed. Beam is assumed as Euler Bernoulli for finite element analysis. He studied the response for fixed-fixed, pined-pined and fixed-pined boundary condition. The study is also carried out for the column of the frame structure. The problem is extended to show the effect on response of the beam structure of various parameter such as velocity of the moving load, spring stiffness, viscous damping and additional stiffness at the conjunction point of beam and column. MATLAB and ANSYS program have been developed and the results were validated. Results are obtained for various alpha values (non-dimensional parameter) and compared for different boundary conditions. Both static and dynamic deflections are experimented. For static case deflection is measured when the load was at the midpoint. It was found that the maximum dynamic displacement occurs at the mid-point of the pinned-pinned beam which is expectable. The additional stiffness increases the rigidity of the frame structure alternately the mode shape shifts up. Damping is also considered and it gives negligible effect on the beam and the frame structure. It is concluded that the longer beam has the smaller natural frequency similarly the longer column has lower natural frequency.

Olsson [3] investigated the dynamic response of beam subjected to a load moving with a constant speed. The vibration response of a simply supported beam with and without elastic foundation to a moving single point load is analyzed performing the finite element vibration analysis and presented analytical and finite element solutions. Olsson investigated taking three approaches i.e. Formation of loading functions, Modal analysis and Transient analysis. In the first case he consider static

Wang & Lin [4] investigated the dynamic response of a multi-span Timoshenko frame structure due to moving load. They used modal analysis technique to study the response.

Zibdeh & Hilal [5] investigated the dynamic behavior of beam structure with different boundary condition traversed by moving load also analyzed the beam structure of different boundary conditions traversed by a moving load. They analyzed the beam structure for different types of motions such as accelerating, decelerating and constant velocity .Also the showed the effect of motion and damping for different boundary condition.

M. Dehestani et. al. [6] Recent development researchers material and constructional technology greatly effected by sudden change in weight of elements of structures. In such conditions inertia effect of moving bodies will not be avoidable. Study of inertia effect helps to investigate response of bridge structures and many engineering application structures that are moving with high speed such as machining processes. They presented analytical and numerical results to find out the response due to moving load. They have considered all the boundary conditions to investigate the best design specifications. Also shown the effect of coriolis component of acceleration related to moving mass of the system. The influence of speed on different boundary conditions is also stated. It concluded that the speed and the inertia of moving mass has direct influence on the response of the structure for all boundary conditions.

Law & Zhu [7] studied the vibration of a damped reinforced concrete beam structure under moving load. They modeled as moving system having four degree of freedom .The moving mass system has linear suspension tyre flexibility. This experiment is carried out for continuous Euler Bernoulli beam simply supported at both the ends. They validate the analytical solution with the experimental results. In their further study they presented indirect approach to to study the response by using a device called structural health monitoring which sensed the response. This technique is applied to bridge-vehicle interaction field. This device processes the information about response of bridge structure to which define the state of bridge.

Karganovin et. al. [8] investigated the response of Timoshenko beam. The beam structure is supported by visco-elastic foundation which subjected to distributed moving load. They presented theoretical analysis to study the response. Effect of frequency and the velocity of dynamic load on displacement is studied. They shown that with the increase of frequency and velocity of moving load on maximum displacement firstly increases and then decreases .

T. Karttunen et. al. [9] studied the dynamic behavior of an elastic cylinder cover using a 1D Pasternak-type foundation model with damping. Moving point load is applied on cylinder cover, which is taken to represent a load resultant due to rolling contact. the natural frequencies, vibration response, wave dispersion relation, total strain energy and dissipation power of the cover are obtained by Analytical expressions . The calculated natural frequencies and modes by the 1D approach are validated to those given by a 2D plane strain finite element model and a satisfactory result is obtained. The critical load speed in the cover is obtained for the un-damped analytical model at a resonance condition. The critical speed is the minimum phase velocity of the waves in the cover. The speeds decrease when damping is included.

Bergman et. al.[10] studied the damaged Euler Bernoulli beam subjected to moving mass. They validated the result both theoretically and experimentally.

Michaltos et. al. [11] investigated the dynamic response of a simply supported uniform beam under a moving load of constant magnitude and velocity. The effect of the mass of the moving load and its velocity and of other parameters is verified using a series solution for the dynamic deflection. A variety of numerical results are obtained to draw important conclusions for the purpose of structural design.

Mehri et al. [12] studied the dynamic response of uniform beams with different boundary conditions subjected to moving load assuming Euler-Bernoulli beam theory. Effects of velocity of mass for different boundary conditions and some other parameters are studied Using dynamic green function and some of the numerical results are compared with the given literatures. An exact and direct modeling technique is formulated for forming beam structures with different boundary conditions, subjected to mass moving at constant speed. The effect of variation in the speed parameters results was given in graphical and tabular form to analyze the dynamic response of beam.

Hamada [13] used double Laplace transform to find a solution for a beam with damping under the action of a mass less load moving at constant speed. He investigated the response damped Euler–Bernoulli uniform beam supported at both the ends. He obtained close solution for dynamic response which is useful for computation of beam stresses. Analysis of Forced vibration is done using double power series method and the coefficient of the series is obtained by the use of Bernoulli polynomials.

Ting et al [14] studied the response of both the beam and the moving mass for dynamic response. He formed an algorithm to study the response including various boundary conditions, for that Initial conditions are necessary to make the problem simpler. He illustrated an example to verify the Results obtained by theoretically and experimentally are well comparable.

P.K.Chatterjee et. al. [15] studied the multi-span continuous Euler Bernoulli beam under dynamic load. They considered the effect of vehicle and bridge surface interaction, also consider the torsion of the bridge structure due to eccentricity of moving vehicle. Iterative procedure is applied to study the response for each time step. A continuous approach is utilized to determine the Eigen functions of the bridge. The method is used to investigate the influence of parameters that are important on the response of the bridge structure. To find the response of continuous beam a continuous approach has been taken. The results were obtained with respect to time and the non-linear effect of moving load for the dynamic response.

Ismail Gerdemeli et. al.[16] investigated the dynamic response of overhead crane beams. The beam was assumed as an Euler Bernoulli beam. Computerized analysis was done in SAP 2000. The response of the crane beam depends on mass and the velocity of the load. The natural frequency of the system changes as the moving load changing its position with time. Depending on the position of the moving mass the system vibration varies.

Trethway et. al.[17] investigated the elastic beam subjected to moving load. The governing equation for the finite element analysis is derived by finite element method which is solved by Runge-Kutta method. This method is much efficient to solve any tome dependent dynamic system motion also gives the best approach to solve the equation with complex boundary condition. They consider the moving system of two foot height having damping and stiffness. In this paper various types of moving load have been presented. They study the dynamic load system having different height moving at constant velocity along the beam. This model well succeed so this method implemented to real field application. Using this technique high speed drilling machine structure is investigated. Effect of various factors like speed, inertia, number of moving load on a single system also considered in their further study. The moving mass inertia has a great effect on the response of structure attributed to dynamic load.

In this thesis dynamic response of beam and frame structure subjected to moving load is studied using finite element method and analytical method. Aim of this thesis is to show the static and dynamic result and compare the results, effect of velocity on the response of the beam and the column of the frame structure is studied.

Chapter 3

3. DYNAMIC RESPONSE OF STRUCTURE

In the middle of the nineteenth century the resonant transverse vibration of highway and railway bridge structure alarmed the engineers to study the dynamic behavior of structure. Collapsed of Stephenson's bridge in London in 1947 attracted the intention of design engineers to research structure subjected to moving load. The intention was to investigate the response of structure under dynamic load and safety of design against dynamic forces , moment and deformation due to moving load. Vibration of structure arises due to motion of vehicle, earth quake, flow of stream and winds. There's various factors need to consider for design safety purpose such as Mass of the moving body and the structure, Inertia of moving mass Twisting of structure due eccentric load.

3.1 BEAM STRUCTURE

Beams are the description of engineering structures. Frames of machines, automobiles and other mechanical structure are composed of beam that are specially designed. All these structures are analyzed in the similar way as beam. Beam is an element that carries vertical and horizontal loads. These loads can be static or dynamic. Static load is due to the external bodies resting on beam and the dynamic load is the result of moving body, due to blow of wind or an earthquake. These loads are transmitted to its supports. Beams are defined by their geometry and the type of material. Now a days usually reinforced concrete, steel or wooden beams are used in constructions.

Beam can be differentiated into three criteria i.e. based on geometry, equilibrium conditions and the types of supports. According to the type of supports beam can be of following types

1. Simply supported beam: Simply supported beam is the simplest structural element in existence. It has pinned support at one end and roller support at the other end. It undergoes shearing or bending, depending on the externally applied load.

2. Cantilever beam: It has fixed support at one end and free at other end.

3. Overhanging beam: The span of the beam extended beyond its supports are called overhanging beam. It is combined feature of simply supported and cantilever.

4. Continuous beam: This type of beam has more than two supports over its length.

5. Fixed beam: In this type of beam both the ends of the beam are fixed.

3.2 FRAME STRUCTURE

Frame structures are the combination of beam and columns. The loads carried by beam of the frame structure are distributed to its column. Columns and slab generally used to support against applied loads, moments and deformation. A rigid frame structure is made up of linear elements, typically beams and columns that are connected to one another at their ends that do not allow any relative rotation to occur between the attached members, although the joints themselves may rotate as a unit.

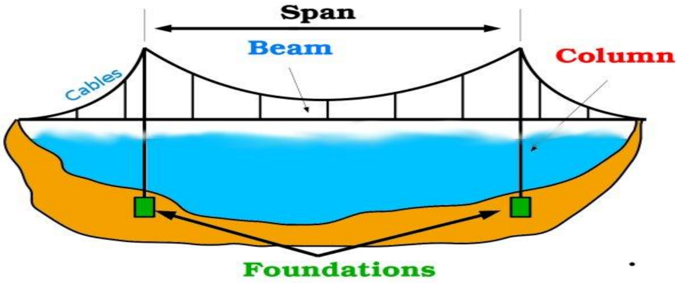

Figure 3.1 Schematic diagram of bridge frame structure

Chapter 4

4. THE FINITE ELEMENT METHOD

It is a simple and effective method to analyze the behavior of regular or irregular, continuous structure for general loadings. In FEM the problem is solved by discretizing the continuous physical structure into simple geometric element. The discrete element is used for the analysis of whole structure. The complex structure is discretized into finite number of elements which are interconnected. The point at which the discrete element are joined is known as node. Each element consist of two nodes. Degrees of freedom are applied at the node for the finite element analysis.

In mathematics, it is a numerical technique use to find the approximate solution of boundary value problems. Complex equation of large domain can easily solve by connecting sub domain equation called as finite element. Computational tool is use to perform the analysis.

4.1 GENERAL PRINCIPLE OF FINITE ELEMENT ANALYSIS:

Discretizing the whole structure into number elements, the element should be of known geometry for the convenience of mathematical formulation. It has several advantage like the complex shape can be represented so the properties of dissimilar material can be included and local effect can be easily captured. Each element represented by sets of equation to the original problem. Total solution is obtained by globalizing the each element for final calculation. Best results can be obtained by maximizing the number of elements.

4.2 General steps of the Finite Element Method

The following general steps discussed below are for structural analysis case.

1. Discretization and choosing element types:

This step includes division of geometry into an equivalent set of finite elements with associated nodes and selecting the best suitable element which resembles the actual physical behavior of the given system to be analyzed. Engineer needs to focus in the matters of selecting the number elements, variation in size and type of elements. For getting best results it is advisable to choose as small elements as possible. One of the major tasks of the engineer is the selection of the appropriate element for a particular problem.

2. Select a Primary variable function:

This step involves selecting a primary variable (displacement) function within each element. The function is defined within the element using the nodal values of the element. Polynomial functions are generally used because they are easy to work within finite element formulation. In case of two dimensional elements, the primary variable function is function of the coordinates in its plane. The functions are expressed in terms of the nodal unknowns.

3. Define relations:

The relations among stresses, strains and displacements are essential for obtaining the equations for each finite element. In the case of one –dimensional deformation, say, in the x direction, we have strain ε_x related to displacement u by

$$\varepsilon_x = \frac{du}{dx}$$

for small strain cases. The definition of material behavior is also important in obtaining acceptable results.

4. Extraction of the element stiffness Matrix and Equations:

The element stiffness matrix and equations are deriving by using the following methods.

Direct Equilibrium Method

According to this method, the stiffness matrix and element equations relating nodal forces to nodal displacements are obtained using force equilibrium conditions for a basic element, along with force or deformation relationships. This method can easily adaptable to line or one-dimensional elements.

Work or Energy Methods

For the extraction of the stiffness and equations for two and three-dimensional elements, the application of work or energy methods are very friendly. The principle of minimum potential energy, the principle of virtual work methods used for derivation of element equations.

Generally, for elastic materials the principle of minimum potential energy is suitable whereas the principle of virtual work can be adopted for any other material.

Methods of Weighted Residuals

The methods of weighted residuals are useful for developing the element equations particularly popular Galerkin’s method. This methods yield the same results as the energy methods wherever the energy methods are applicable. They are especially useful when a functional such as energy method is not readily applicable.

5. Assembling the Element equations and Apply boundary conditions:

In this step, the equilibrium equations of nodes that are obtained in previous step are combined into the global nodal equilibrium equations. One more direct method of superposition, whose basis is nodal force equilibrium, can be used to obtain the global equations.

The global equation can be written in matrix form as

$$\{F\} = [K]\{\delta\}$$

Where $\{F\}$ represents the Force vector, $[K]$is the total stiffness matrix $\{\delta\}$ is the vector of generalised displacements.

At this stage, the global stiffness matrix $[K]$is a singular matrix because its determinant is equal to zero. To remove this we need to call upon certain boundary conditions in order to avoid to the movement of the structure as rigid body.

6. Solve for the primary unknowns:

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_n \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{1n} \\ K_{21} & K_{22} & K_{2n} \\ K_{n1} & K_{n2} & K_{nn} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_n \end{Bmatrix}$$

These general equations can be solved for the primary unknowns by using an elimination method or an iterative method. The primary variables are different for various problems. In case of the structural problem the unknown primary variable is displacement.

7. Solve for secondary unknowns:

In this step secondary unknowns are determined by using the displacement equations which are already obtained from previous step. Commonly strain and stress, shear force and moments are secondary unknowns for structural problem, are determined by using mathematical techniques.

8. Interpret the results:

The results obtained in previous step need to analyze for use in the design or analysis process. For better design and to avoid the failure of the structure it is important to determine the locations in the structure where large deformations and stresses is occur. Postprocessor computer programs help the user to interpret the results by displaying them in graphical form.

4.3 APPLICATIONS OF THE FINITE ELEMENT METHOD

It is an effective tool to analyze both structural and non-structural problems. Structural analysis include vibration, buckling and stress analysis the stress concentration problems associated with fillet, holes and other changes in the geometry of the body. Non-structural analysis include fluid flow, transfer of heat and the distribution of a electric or magnetic potential. It can be applied to solve biomechanical engineering problems such as analyses of human spine, skull, hip joints, heart and eye etc.

4.4 LIMITATIONS OF THE FINITE ELEMENT METHOD

In spite of many advantages it has certain drawbacks also which are as follows

- Stress values depend on the size of mesh.
- In some cases the approximations used may not provide accurate results.
- For vibration and stability problems the cost of analysis by FEA is prohibitive**.**

Chapter 5

5. FINITE ELEMENT ANALYSIS OF BEAM ELEMENT

Beam element:

A uniform cross section beam element is considerd to form the finite element matrices as shown in figure given below. Euler Bernoulli theory is assumed for analysis of beam. In the figure given below represents the element of the beam having length l and x represent the length at any instant of time from the starting of node to the point at which the load is acting.

In the figure 5.1 **E,** $\boldsymbol{\rho}$ and **I** are the modulus of elasticity, density and constant moment of inertia of the beam respectively. For each node two degree of freedom is considered for finite element analysis.

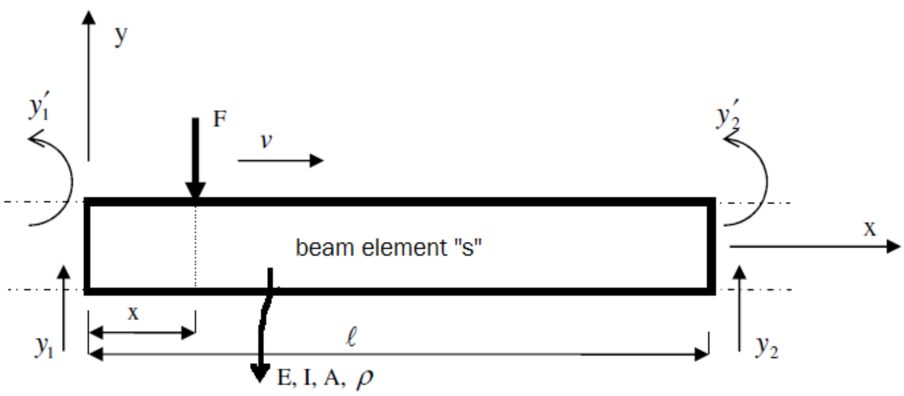

Fig 5.1 Beam element

Strain energy and the kinetic energy of the beam element is considered to formulate constituting the finite element matrices.

Strain energy for the considered single element is give in the expression below:

$$S.E. = \tfrac{1}{2}\int_0^l EI\left\{\frac{\partial^2 y}{\partial x^2}\right\}^2 dx \tag{1}$$

Where E and I are the modulus of elasticity and the inertia of the beam. Kinetic energy of a single element is given as

$$KE = \int_0^L \frac{\rho A}{2}\left(\frac{\partial y}{\partial t}\right)^2 dx \tag{2}$$

For the transverse displacements a cubic function, y(x,t) has assumed,

$$y(x,t) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \tag{3}$$

Where a_0, a_1, a_2, a_3 are the functions of " t" which can be obtained by applying boundary conditions at the corresponding nodes, Rao(1995)[19] represented the shape function as given below

$$N_1(x) = 1 - 3\left(\frac{x}{l}\right)^2 + 2\left(\frac{x}{l}\right)^3 \tag{3.1}$$

$$N_2(x) = x - 2l\left(\frac{x}{l}\right)^2 + l\left(\frac{x}{l}\right)^3 \tag{3.2}$$

$$N_3(x) = 3\left(\frac{x}{l}\right)^2 - 2\left(\frac{x}{l}\right)^3 \tag{3.3}$$

$$N_4(x) = -l\left(\frac{x}{l}\right)^2 + l\left(\frac{x}{l}\right)^3 \tag{3.4}$$

Applying boundary condition on the transverse displacement cubic function

$$y(x,t) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \tag{4}$$

At x=0, $y_1 = a_0 \; \dot{y}_1 = a_1$

At x= l, $y_2 = a_0 + a_1 l + a_2 l^2 + a_3 l^3 \; \dot{y}_2 = a_1 + 2a_2 l \;\; + 3a_3 l^2$

The above solution can be represented in the matrix form

$$\begin{bmatrix} y_1 \\ \dot{y}_1 \\ y_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & l & l^2 & l^3 \\ 0 & 1 & 2l & 3l^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Which can be written as

$$\{y\} = [C]\{a\} \tag{4.1}$$

Or $\{a\} = [C]^{-1}\{y\}$ (4.2)

We know from the equation (4)

$$y = [x]\{a\} = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = [x][C]\{y\} \tag{4.3}$$

$$\frac{\partial y}{\partial t} = [x][C]^{-1}\{\dot{y}\} \text{and} \left(\frac{\partial^2 y}{\partial t^2}\right) = \left\{\frac{\partial y}{\partial t}\right\}^T \left\{\frac{\partial y}{\partial t}\right\} \tag{4.4}$$

$$K.E. = \frac{1}{2}\int_0^l \rho A [C]^{-1}\{\dot{y}\}^T \begin{bmatrix} 1 \\ x \\ x^2 \\ x^3 \end{bmatrix} \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} [C]^{-1}\{\dot{y}\} dx \tag{5}$$

Solving the above equation the mass matrix can be obtained as follow:

$$M = \frac{\rho AL}{420} \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{6}$$

From equation (1) strain energy given an as follows:

$$S.E. = \tfrac{1}{2}\int_0^l EI\left\{\frac{\partial^2 y}{\partial x^2}\right\}^2 dx$$

twice derivative of equation (4.3) will give acceleration as

$$\left(\frac{\partial^2 y}{\partial x^2}\right) = \begin{bmatrix}0 & 0 & 2 & 6x\end{bmatrix}[C]^{-1}\{y\} \tag{7}$$

$$S.E. = \frac{1}{2}\int_0^l EI\{y\}^T[C]^{-T}\begin{bmatrix}0\\0\\2\\6l\end{bmatrix}\begin{bmatrix}0 & 0 & 2 & 6x\end{bmatrix}[C]^{-1}\{y\} \tag{8}$$

Equation (8) give the stiffness matrix as follows :

$$K = \frac{EI}{l^3}\begin{bmatrix}12 & 6l & -12 & 6l\\ 6l & 4l^2 & -6l & 2l^2\\ -12 & -6l & 12 & -6l\\ 6l & 2l^2 & -6l & 4l^2\end{bmatrix} \tag{9}$$

Equation 6 and 9 are the mass and stiffness matrix of a single beam element. In this analysis beam structure is constitute of 10 elements, so overall mass and stiffness matrices can be formed by assembling 10 element.

5.1 ANALYSIS OF FRAME ELEMENT

For finite element analysis of frame element displacement is assumed to be linear for longitudinal vibration along the axial direction.

$$u(x,t) = e(t) + f(t)$$

Applying initial conditions on the above displacement equation the corresponding shape function for the frame element can be obtained as given below.(Rao,1995)

$$N_5 = \left(1 - \frac{x}{l}\right) \qquad (10)$$

$$N_6 = \frac{x}{l} \qquad (11)$$

For the beam element two degrees of freedom is considered for each node. Bending and torsion effect on the frame element is neglected.

A transformation matrix is used to form the element mass matrix and the element stiffness matrices of the frame. Frame constitute of two columns and a beam. The beam and each column is constitute of ten elements of equal size. Transformation matrix is multiplied with the mass and the stiffness matrices of the beam element to constitute the overall mass and the stiffness for the frame structure. Transformation matrix is different for both the columns. Transformation matrix is given as follows

$$T = \begin{bmatrix} \cos\theta & \sin\sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

And the overall mass and the stiffness can be formed as $[K] = [T]^T [K][T]$ (12)

and the mass matrices can be formed as $[M] = [T]^T [M][T]$ (13)

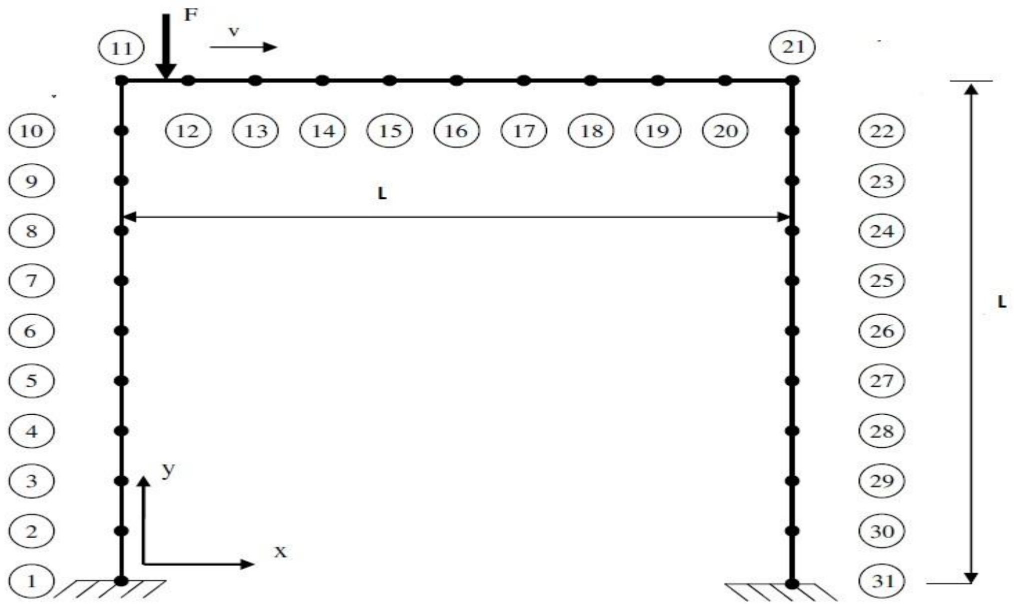

Fig. 5.2 nodal representation of frame structure

The frame structure is discrete into equal size thirty element for finite element analysis as shown in the above figure. The above structure consist 31 nodes. Node 1 to 10 represents column 1, node 11 to 21 represents the beam over which the load moves and the node 22 to 31 represents column 2.

Modeling of the frame is given below. In the transformation matrix the θ equal to 90 degree for column 1 and 270 degree for the column 2.

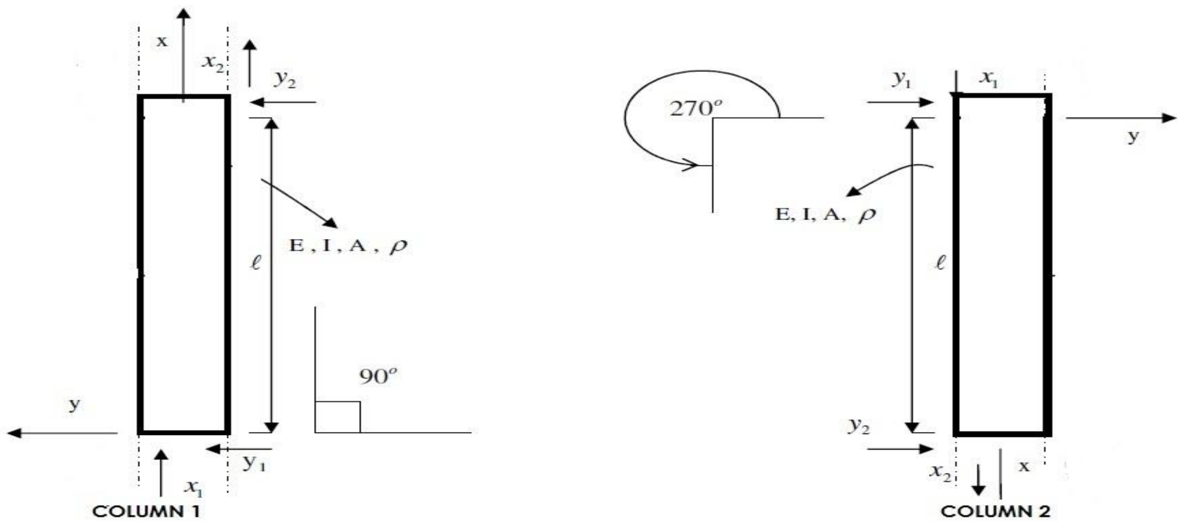

Fig 5.3 Modeling of Column 1 and column 2

Chapter 6

MATHEMATICAL FORMULATION

6.1 EQUATION OF MOTION FOR FORCED VIBRATION SYSTEM

Equation of motion for multi degree of freedom system is given as

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=\{F(t)\} \tag{14}$$

In the above equation $\{u(t)\}$ is the nodal displacement vector, [M] and [K] are the overall mass and stiffness matrices of the beam structure. {F (t)} is the external force vector.

6.2 EXPRESSIONS FOR EXTERNAL FORCE VECTOR:

The external force vector can be represented by equivalent nodal forces. According to Clough and Penzien [18] for three degree of freedom system the external force vector takes the following form:

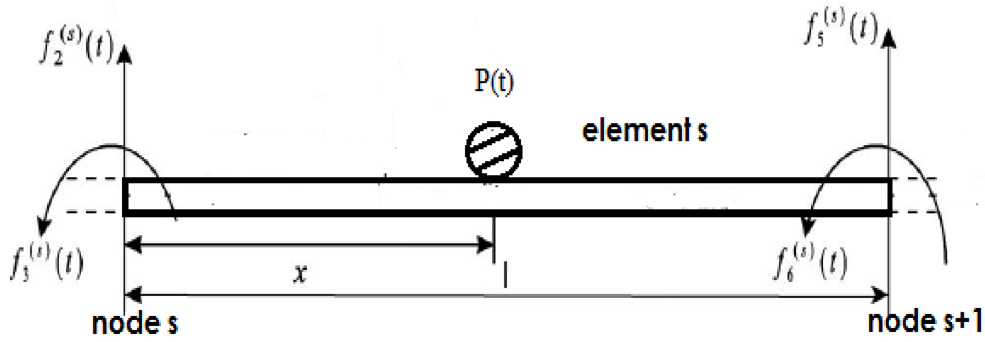

Fig 6.1 nodal forces acting on beam element

$$\{F(t)\}=\{0....f_1^{(s)}(t)\,f_2^{(s)}(t)\,f_3^{(s)}(t)\,f_4^{(s)}(t)\,f_5^{(s)}(t)\,f_6^{(s)}(t).......0\} \tag{14a}$$

In the fig $f_1^{(s)}(t) f_2^{(s)}(t) f_3^{(s)}(t)$ and $f_4^{(s)}(t) f_5^{(s)}(t) f_6^{(s)}(t)$ are the nodal forces at node s and s+1 respectively. $f_1^{(s)}$ and $f_4^{(s)}$ are the nodal forces along x-axis, $f_2^{(s)}$ $f_5^{(s)}$ are the forces along y-axis and $f_3^{(s)}$ $f_6^{(s)}$ are the rotation about the z-axis on the corresponding nodes. In this study a two degree of system is consider so the axial nodal forces along $f_1^{(s)}$ and $f_4^{(s)}$ are taken to be zero.

$$\{F(t)\} = \{f^s(t)\}$$

$$\{f^s(t)\} = [f_2(t) f_3(t) f_5(t) f_6(t)]^T = P\{N\}$$

$$\{N\} = [N_1 N_2 N_3 N_4]^T$$

Where $N_1 N_2 N_3 N_4$ are the shape functions. According to (Rao,1995) [19] shape functions are represented as given below.

$$N_1 = 1 - 3\xi^2 + 2\xi^3$$

$$N_2 = (\xi - 2\xi^2 + \xi^3)l$$

$$N_3 = 3\xi^2 - 2\xi^3$$

$$N_4 = (-\xi^2 + \xi^3)l$$

$$\xi = \frac{x}{l}$$

It should be noted that "l" is the length of the beam element and "x" is the distance of moving point load along the element from the moving end to the point of application as shown in the figure (6.1). The moving load can be expressed by applying forces and moments which are the function of time to the entire nodes of the finite element model.

6.2(a) A MOVING POINT LOAD

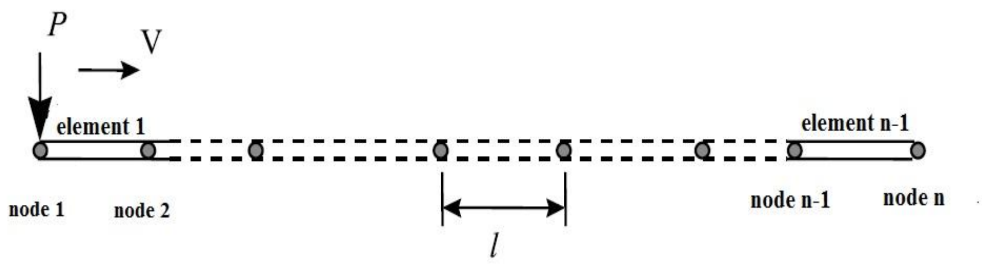

Fig 6.2 a point load moving from node 1 to node n

In this analysis the moment is not taken into consideration. As shown in fig(6.2), a concentrated force P along the beam from node 1 to n with a velocity V. the beam is composed of n-1 elements. Considering m time step with time interval Δt the total time or maximum time is given by

$$t_{\max} = m\Delta t$$

$$\{F_i^y\} = [F_1^y F_2^y F_3^y \ldots\ldots F_k^y]^T \tag{14b}$$

Where i represent the node number which varies from 0 to k.

Taking initial condition

At time t = 0

Assuming the moving mass is stationary and acceleration zero at node 1 we may obtain

$$\{F_1^y\} = P$$

At time t(t$\neq$ 0)

$$\{F_s^y\} = PN_1$$

$$\{F_{s+1}^y\} = PN_3$$

For i = 1 to k, $(i \neq s, s+1)$

$$\{F_i^y\} = 0$$

Where i represent the node number which varies from 0 to k.

At any time $t = r\Delta t$ (r =1 to m) then the position of the moving point load is given by

$x_p(t) = Vr\Delta t$; so at any time the element number over which the load is acting can be found out from $s = \frac{x_p(t)}{l} + 1$, that numerical value of $\frac{x_p(t)}{l}$ be in integer .

The shape function of node s and s+1 is obtain by the following expression:

$$N_1 = 1 - 3\xi^2 + 2\xi^3$$

$$N_3 = 3\xi^2 - 2\xi^3$$

Where $\xi = \dfrac{x_p(t) - (s-1)l}{l}$

Noting that $x_p(t)$ is the variable distance between the moving load from the left end of the 's'th element. Putting above instantaneous forces the overall external force vector can be obtain by the equation given as

$$\{F_i^y\} = [F_1^y F_2^y F_3^y ......F_k^y]^T$$

6.3 MOVING LOAD PROBLEM

For normal mode analysis procedure, the displacement at any node is given by

$$\{u(t)\} = [\phi]\{q(t)\} \tag{15}$$

$\{q(t)\}$ is the nodal displacement vector and $[\phi]$ is the mode shape matrix which consist of n independent nodal vectors as given below.

$$[\phi] = [\{\phi_1\}\{\phi_2\}\{\phi_3\}\{\phi_4\}\{\phi_5\}............\{\phi_n\}]$$

$[\phi]$ consist of information about horizontal, vertical and rotational deformation. The differentiating Eq. (15) velocity and acceleration vector can be obtained as

$$\{\dot{u}(t)\} = [\phi]\{\dot{q}(t)\} \tag{16}$$

$$\{\ddot{u}(t)\} = [\phi]\{\ddot{q}(t)\} \tag{17}$$

Neglecting damping and substituting the values of Eq.(15), (16) and (17) into equation (14) it is obtained.

$$m_n\ddot{q}_n(t) + k_n q_n(t) = f_n(t) \tag{18}$$

$$m_n = \{\phi_n\}^T [M]\{\phi_n\}$$

$$k_n = \{\phi_n\}^T [K]\{\phi_n\}$$

$$f_n(t) = \{\phi_n\}^T \{F(t)\}$$

Rearranging the equation Eq. (18)

$$m_n\ddot{q}_n(t) = f_n(t) - k_n q_n(t) \tag{18a}$$

$$\ddot{q}_n(t) = \frac{f_n(t) - k_n q_n(t)}{m_n}$$

$$\ddot{q}_n(t) = \frac{f_n(t) - \omega^2{}_n \; m_n q_n(t)}{m_n} \tag{19}$$

Neglecting damping

$$\ddot{q}_n(t) = \frac{f_n(t) - \omega^2{}_n \; m_n q_n(t)}{m_n} \tag{20}$$

Inserting the value of eq.(20) in the Eq.(17) $\{\ddot{u}(t)\}$ gives the acceleration along transverse direction. For element "s" the point between node "s" and "s+1" the vertical acceleration can be found out by linear interpolation.[18]

$$a^y{}_m(t) = a^y{}_s(t) + \frac{(x_m(t) - (s-1)l)(a_{s+1}(t) - a^y{}_s(t))}{l} \tag{21}$$

$a^y{}_s(t)$ is the acceleration of (3(s+1)+2) th element which can be obtain from $\{\ddot{u}(t)\}$ as given by equation (17).

At time t=0

$a^y{}_s(t) = 0$,

$\{q(t)\} = 0$,

$\{\dot{q}(t)\} = 0$

then $\{\ddot{q}(t)\}$ is determined by solving equation (20). Hence the value of acceleration is obtained from equation (21). The value of $\{q(t)\}$, $\{\dot{q}(t)\}$ is found out from equation (17a) by numerical simulation. By substituting the value of $\{q(t)\}$ in the equation (15) gives the dynamic response of structure. By solving following the above procedure the new values of $a^{y}{}_{s}(t)$, $\{q(t)\}$ and $\{\dot{q}(t)\}$ is obtained, these values are taken as initial value for next calculation.

NODAL DISPLACEMENT

The nodal displacement of structure is obtained by linear interpolation [20]

$$w = w_s^y + \frac{(x_m(t) - (s+1)l)(w_{s+1}^y(t) - w_s^y(t))}{l} \tag{22}$$

Where w_s^y and $w_{s+1}^y(t)$ are the $\{u(t)\}$ of (3(s-1)+1)th and (3s+2)th element given by equation (15)

SOLUTION OF EQUATION OF MOTION

Table 1: Result obtain for Natural frequency of the beam by taking following properties and geometry.

Length(L) In meter	Area(A=b x w) In meter	Density(ρ) Kg/m^3	Modulus of elasticity(E) in G Pa
1	**.01 x .01**	**7860**	**210**

Material is assumed isotropic. The same properties are considered for analysis of frame structure. To know the effect due to change in stiffness on the response of the frame structure two different stiffness considered such as k1=60000 N/m and k2=400000 N/m.

APPLICATION OF MOVING LOAD

The transverse point load P is moving with uniform velocity, ($V = L/\tau$) along the beam. Where τ is the total time taken by moving load over beam having length L. time taken to reach "i" th node is $\tau_i = x_i / V$,where x_i is the location of ith node from starting point of moving load. A constant load P=150 N is taken.

Chapter 7

7.1 NUMERICAL RESULTS

Table 2: Natural frequency of beam with different boundary conditions:

Sl.no.	Clamped-Clamped Natural frequency (Hz)	Pinned-Pinned Natural frequency (Hz)	Clamped-pinned Natural frequency (Hz)
	MATLAB	MATLAB	MATLAB
1	53.134021	23.43854	36.61583
2	146.49908	93.76356	118.6771
3	287.40531	211.0582	247.7498
4	475.88014	375.6354	424.2475
5	713.06762	588.2723	649.1062
6	1000.8781	850.4827	924.0469

Table 3: Natural frequency of frame element for clamped-clamped condition.

Sl.no.	(Frame without stiffness) Natural frequency (Hz)	(For k1=60000) Natural frequency (Hz)	(For k2=400000) Natural frequency (Hz)
	MATLAB	MATLAB	MATLAB
1	6.617595	13.8009	14.66242
2	24.71361	44.1372	52.48354
3	53.13402	53.6639	60.53828
4	54.12708	101.0558	118.2718
5	94.50837	145.4388	146.9696
6	146.4991	212.5705	221.8313

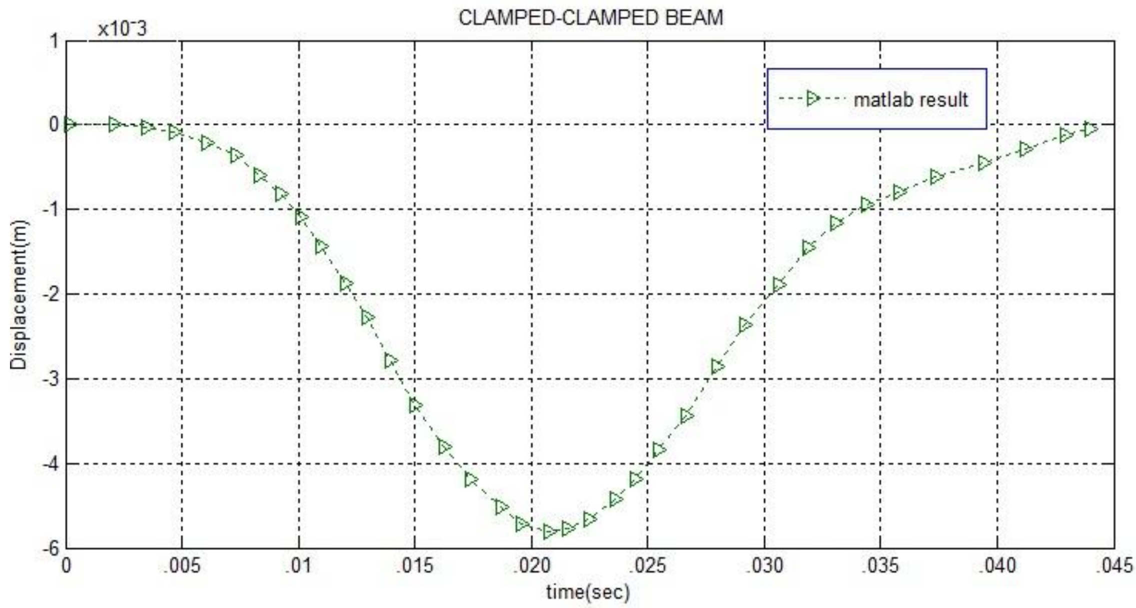

Figure 7.1 Mid point displacement of clamped-clamped beam of load moving with velocity 30 m/s.

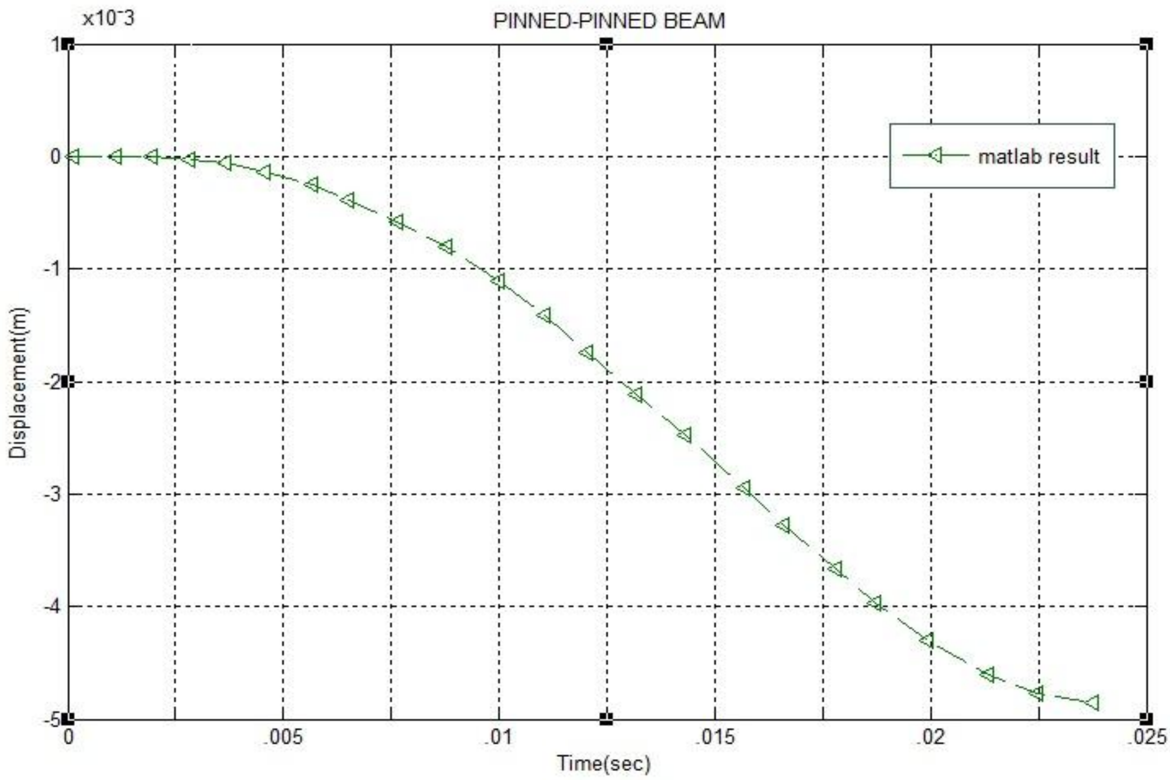

Figure 7.2 Mid point displacement of pinned-pinned beam of load moving with velocity 45 m/s

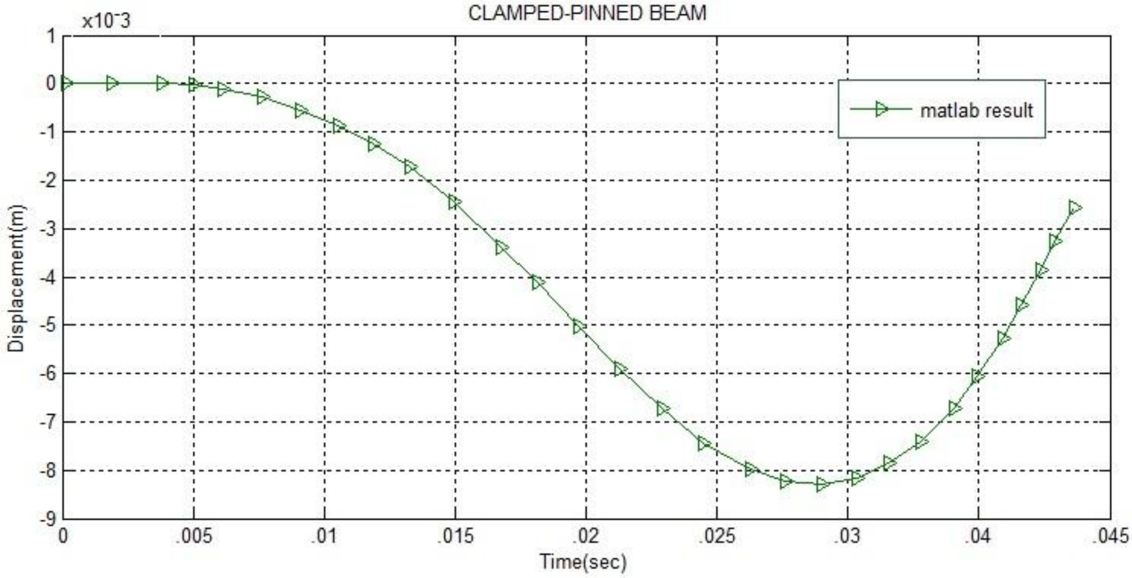

Figure 7.3 mid point displacement of clamped-pinned beam of load moving with velocity 30 m/s.

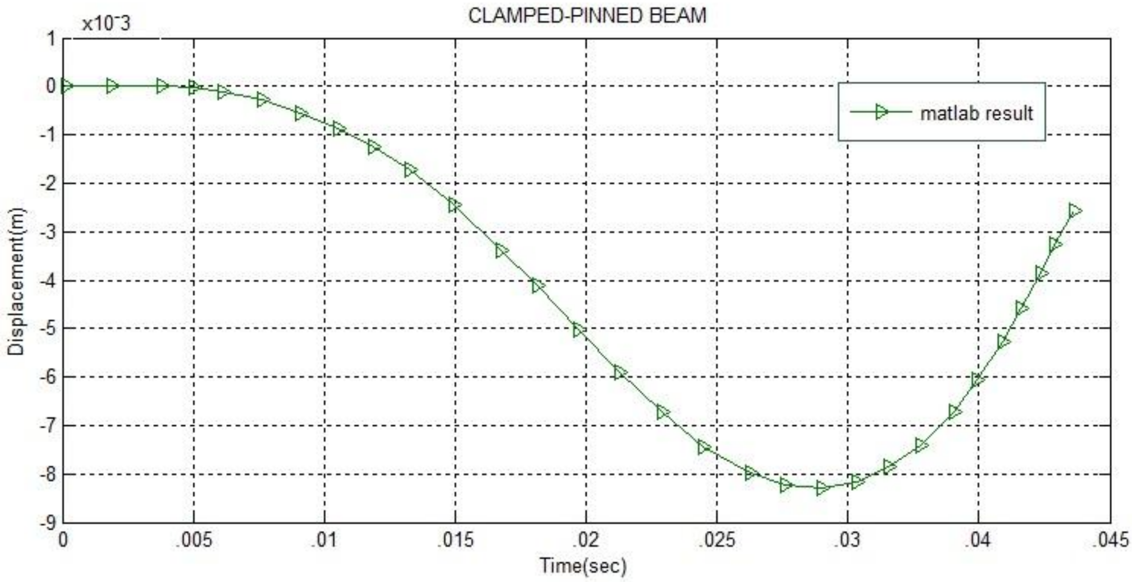

Figure 7.4 mid point displacement of clamped-clamped frame of load moving with velocity 33 m/s

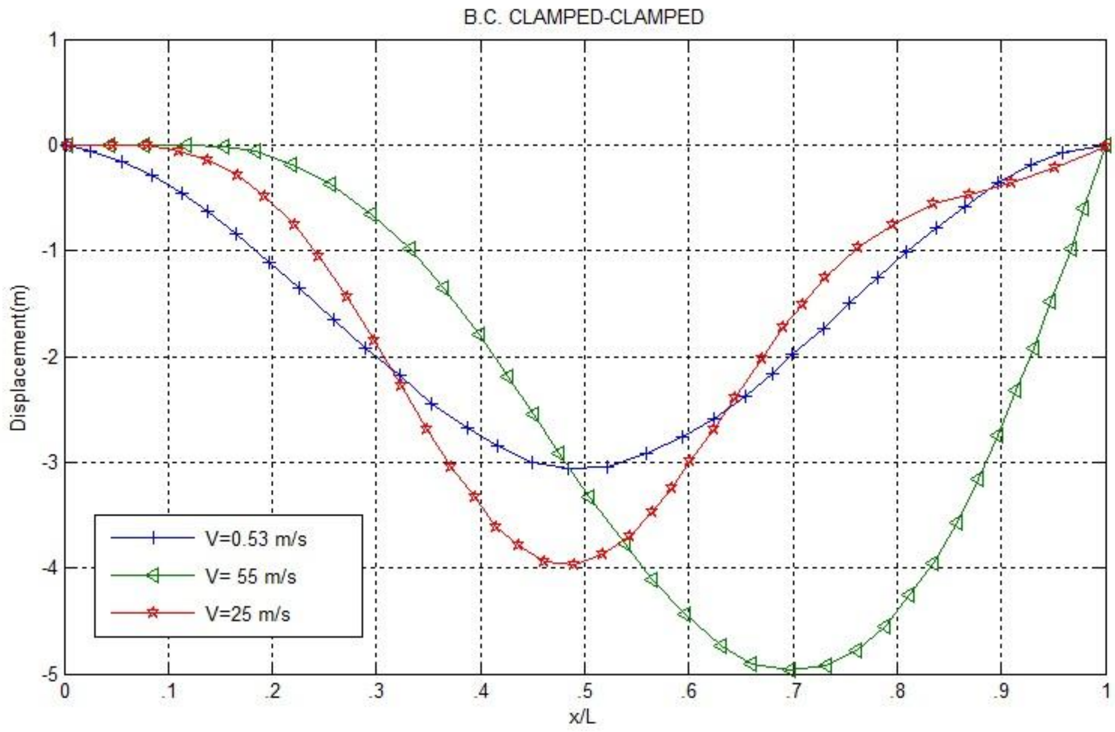

Figure 7.5 dynamic displacement of mid-point of clamped-clamped beam versus the position of moving load for different speeds.

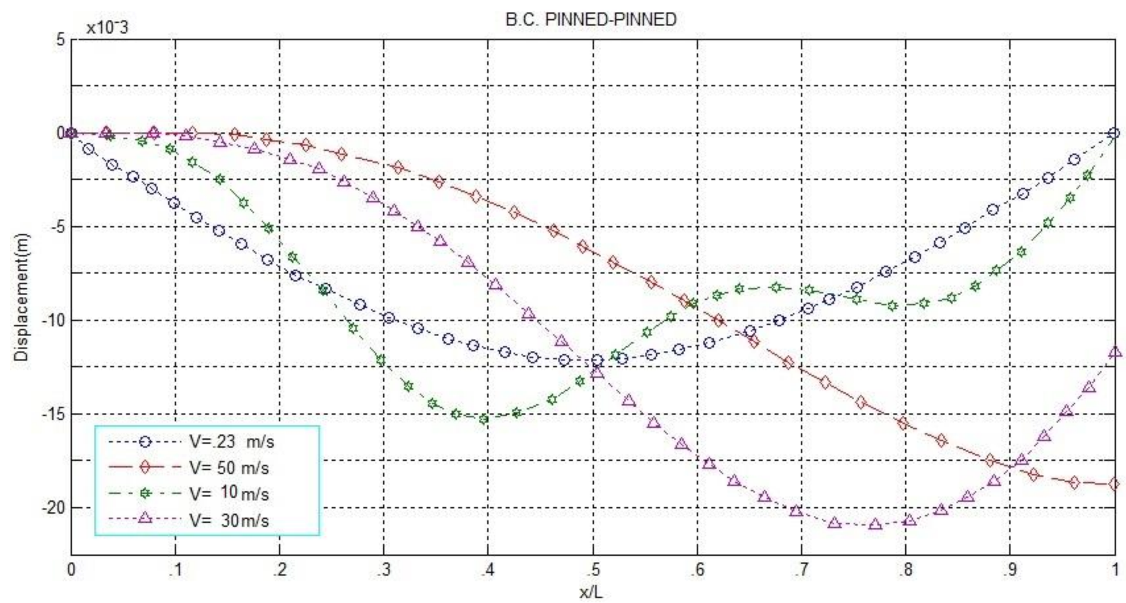

Figure 7.6 dynamic displacement of mid-point of pinned-pinned beam versus the position of moving load for different speeds.

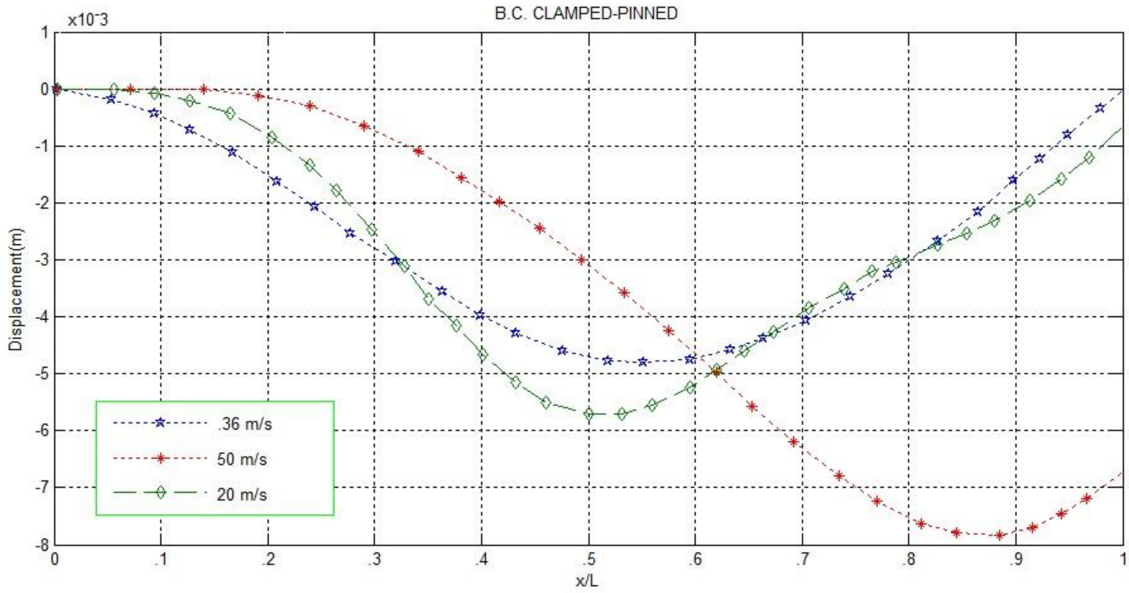

Figure 7.7 dynamic displacement of mid-point of clamped-clamped beam versus the position of moving load for different speeds.

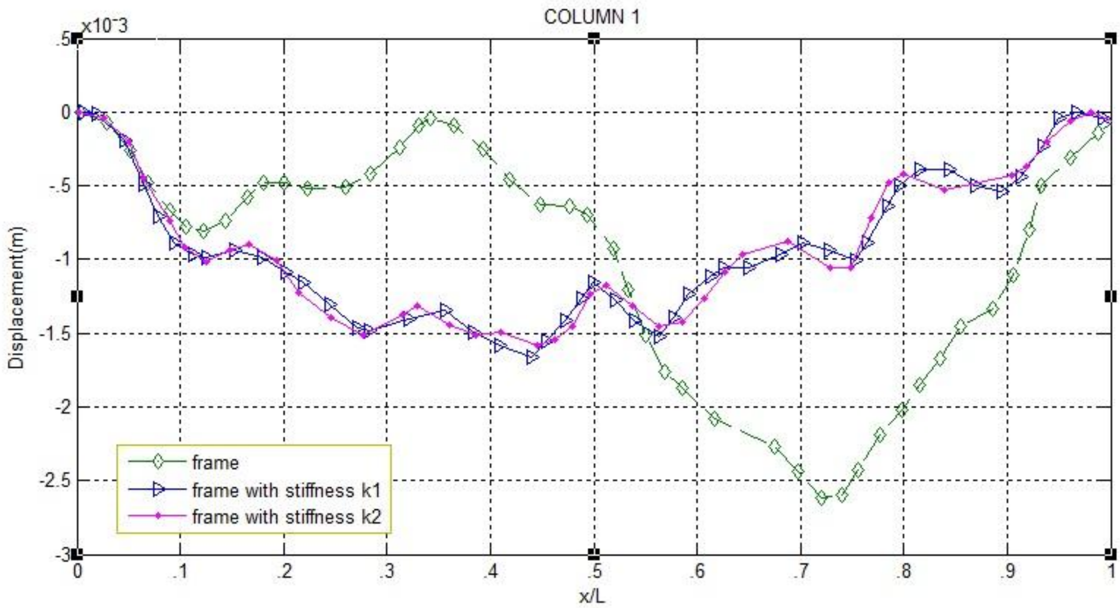

Figure 7.8 Comparison of midpoint displacement of the column 1 of the frame structure with stiffness k1=60000 and k2=400000 (v= 5m/s).

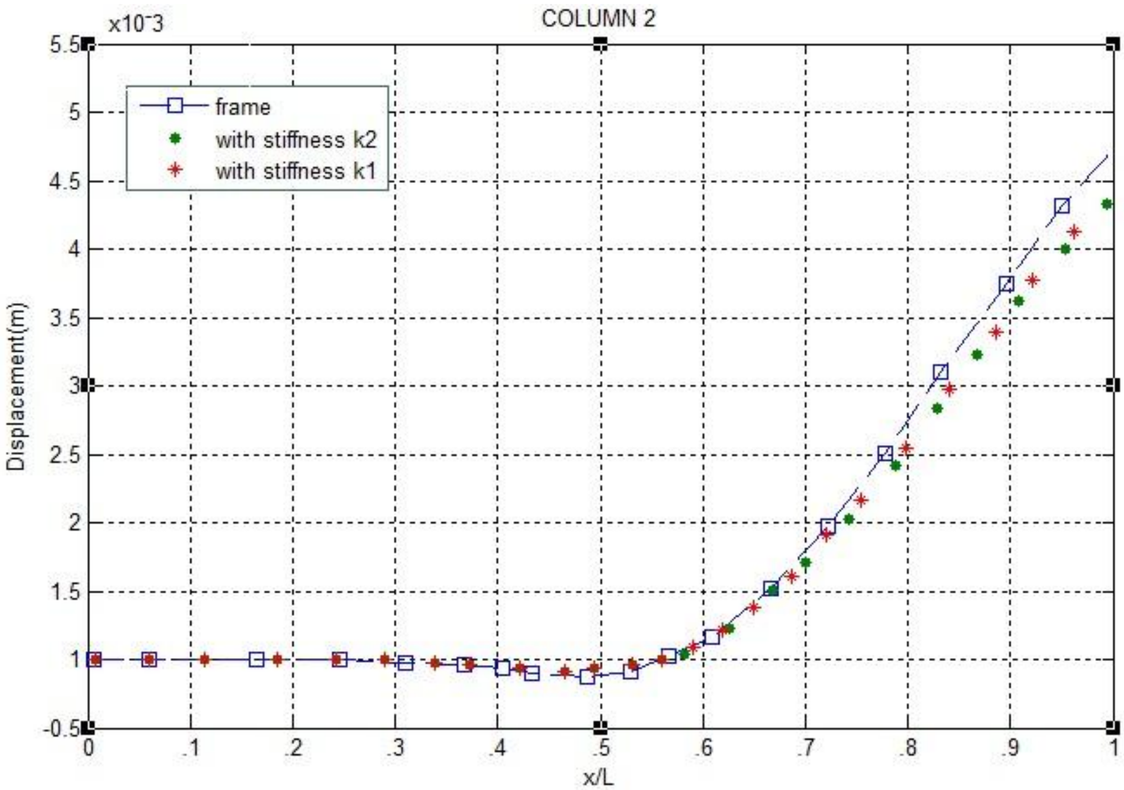

Figure 7.9 Comparison of midpoint displacement of the column 2 of the frame structure with stiffness k1=60000 and k2=4000000 (v= 5m/s).

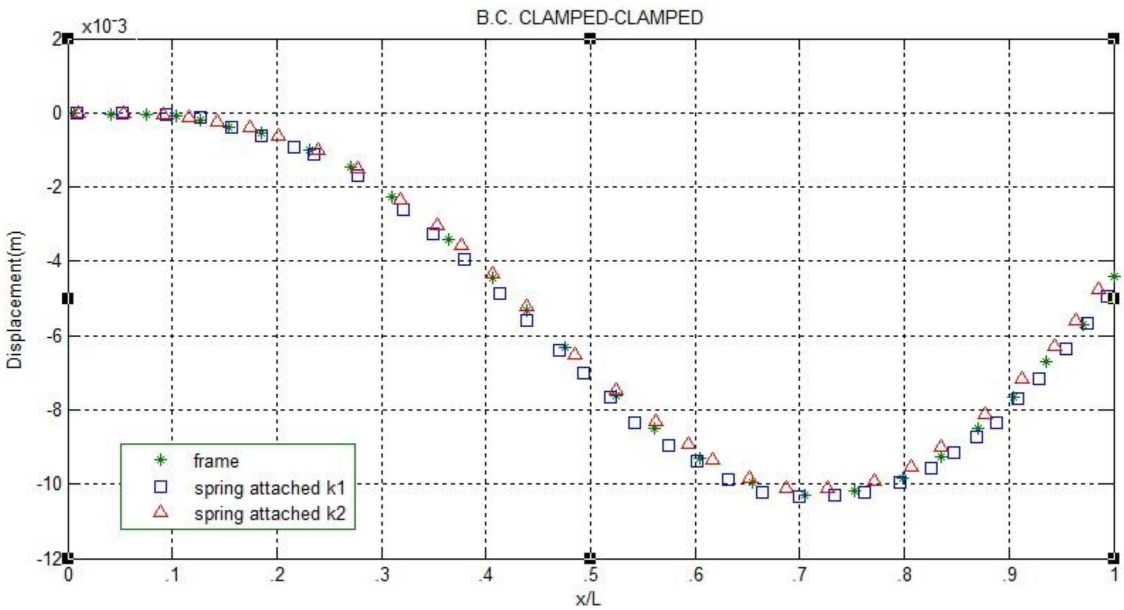

Figure 7.10 effect of stiffness on the mid-point displacement of beam of frame structure (v=35 m/s).

Dynamic magnification factors of Beam for different velocities.										
Velocity(m/s)	5	10	15	20	25	30	35	40	45	50
Clamped-Clamped	1	0.988	1.034	1.091	1.296	1.520	1.532	1.576	1.595	1.612
Pinned-Pinned	1.051	1.089	1.123	1.051	1.247	1.403	1.549	1.612	1.677	1.713
Clamped-Pinned	1.017	1.042	1.079	1.033	1.216	1.354	1.435	1.465	1.596	1.642

Table 4 dynamic magnification factors at the mid-point of the beam for different velocities.

Dynamic magnification factors of Beam for different velocities.										
Velocity(m/s)	50	55	60	65	70	75	80	85	90	100
Clamped-Clamped	1.645	1.656	1.499	1.576	1.533	1.481	1.443	1.401	1.367	1.316
Pinned-Pinned	1.711	1.726	1.722	1.709	1.690	1.678	1.633	1.618	1.534	1.432
Clamped-Pinned	1.623	1.653	1.663	1.645	1.623	1.613	1.596	1.543	1.511	1.336

Table 5 dynamic magnification factors at the mid-point of the beam for different velocities.

Chapter 8

DISCUSSION

In present work mathematical formulation for the beam and frame is done by using FEA. The results are obtained using MATLAB software and validated with the previous research work. Results obtained are well satisfactorily validated with the previous work done on moving load. The dynamic displacement is found out at the mid-point for various boundary conditions. Results are obtained for beam and the frame structure. The effect of additional stiffness and speed of the moving load are also investigated.

The natural frequencies for the beam and frame structure are found out for different boundary conditions using MATLAB program. The first six natural frequencies are obtained given in Table 2.

Displacement versus time of beam and frame structure for clamped-clamped, pinned-pined and clamped-pinned boundary conditions are shown in figures 7.1 to 7.4.

Figure 7.1 present results for mid-point displacement of the clamped-clamped beam under the load moving with velocity 30 m/s. The result is compared with the previous study (Whittaker and Cartmell, 2000)[20] It agrees well with it. Figure 7.2 shows that the displacement versus time at the midpoint of the pinned-pinned beam (V=45 m/s). It is noticed that when the load was at the beginning node there's no significant response at the midpoint but the response changing with time as the load moves along the beam. As compared to clamped-clamped beam, the dynamic displacement is maximum for pinned-pinned case. Figure 7.3 shows the displacement at the midpoint for clamped-pined condition. The results are well agrees with the previous research (S. Hugul [2]).

The dynamic response for the frame structure is investigated by considering clamped at both the ends. Figure 7.4 shows the response of clamped frame structure. One can compare the result obtain for beam and the frame structure, it can be seen that the dynamic displacement is maximum at the midpoint in case of frame structure as compared to the beam structure. The response is validated with previous work done by

(JJ.whittaker, 2000) [20].

Figures (7.5-7.7) show the dynamic displacements at the mid-point of the beam with respect to the position of moving load for clamped-clamped, pinned-pinned and clamped-pinned conditions. One can compare the effect of speed of the moving load on the dynamic response. It is seen that the time at which maximum displacement observed shifts right with the increasing of speed irrespective of the boundary conditions. For low velocity case the response due to moving load is negligible, at this condition the response curve is close to the static displacement at the mid-point of the beam for all boundary conditions [2].

Figure 7.8 and 7.9 shows the midpoint displacements of column 1 and column 2 of the frame structure. In both the figures the effect of stiffness k1=60000 N/m and k2=400000 N/m are shown. It is seen that for a given speed the additional stiffness at the junction of the column and beam of the frame structure has more effect on the dynamic response of mid-point of column 1, the maximum displacement is reduced. On the other hand the spring stiffness has no contribution on the mid-point dynamic displacement of column 2.

Figure 7.10 shows the mid point displacement of the beam of the frame structure when the load is moving at a speed of 35 m/s. it is seen that there is no worthy effect of additional spring stiffness on the mid-point displacement of beam of the frame structure.

The dynamic magnification factor for clamped-clamped, clamped-pinned and pinned-pinned beam is given in the Table 4 and 5. Dynamic magnification factor is the ratio of dynamic displacement to the maximum static displacement. For pinned-pinned case the dynamic magnification factor is greater as compared to clamped-clamped, clamped-pinned.

Chapter 9

9.1 CONCLUSIONS

Euler Bernoulli beam theory is assumed for vibration analysis. Here the moving load analysis is extended to analyze the vibration of frame structure with spring attached to it. The dynamic response for beam and frame structure is studied and following conclusions are drawn:

1. The position of moving load at which maximum displacement is observed is shifts right from left end of the beam with the increase of speed of the moving load for clamped-clamped, pinned-pinned and clamped-pinned boundary condition.
2. The maximum dynamic displacement is observed for pinned-pinned beam compared to clamped-clamped and clamped-pinned boundary conditions.
3. The dynamic magnification factor is found out and the it is observed that for pinned-pinned boundary condition the values of the dynamic magnification factor are greater as compared to clamped-clamped and clamped-pinned.
4. Addition of stiffness at conjunction point of the beam and column of the frame structure increases the rigidity and its natural frequency.

9.2 SCOPE FOR FUTURE WORK

- The present research work can be extended to Timoshenko beam.
- Effect of acceleration of a moving mass over a structure can be studied, which has high impact on the dynamic response of the structural system. It can give engineers some advantages to make a more realistic modeling of structural systems under accelerating mass motion than classical methods that omit inertial effects of accelerating mass.
- There are situations where more than one load travels over a structure such as railway bridges, highway bridges etc. Response of beams to such types of moving load is research worthy.

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